The Classically conformal B-L Extended Ma model

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The Classically conformal B-L extended Ma model solves 2 hierarchy problems

① Neutrino mass scale and EW scale
⇒Ma model

② EW(TeV) scale and Planck scale
⇒Classically conformal symmetry

Neutrino mass and Ma model

Neutrino hierarchy

Quark sector

$$m_u = 2.3 \times 10^{-3}\ GeV,\ m_c = 1.3\ GeV,\ m_t = 173\ GeV$$
$$m_d = 4.8 \times 10^{-3}\ GeV,\ m_s = 9.5 \times 10^{-2}\ GeV,\ m_b = 4.2\ GeV$$

Lepton sector

$$m_e = 5.1 \times 10^{-4}\ GeV,\ m_\mu = 0.11\ GeV,\ m_\tau = 1.8\ GeV$$

$$m_\nu \sim 10^{-11}\ GeV$$

Neutrino masses are very light

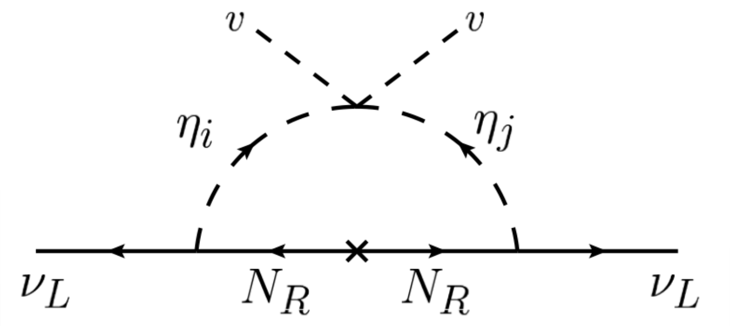
Ma model

E. Ma, Phys.Rev.D73, 077301 (2006)

Ma model is minimal radiative see-saw model with DM

Fermion	L_L	e_R	N_R
$(SU(2)_L, U(1)_Y)$	$(\mathbf{2}, -1/2)$	$(\mathbf{1}, -1)$	$(\mathbf{1}, 0)$
Z_2	$+$	$+$	$-$

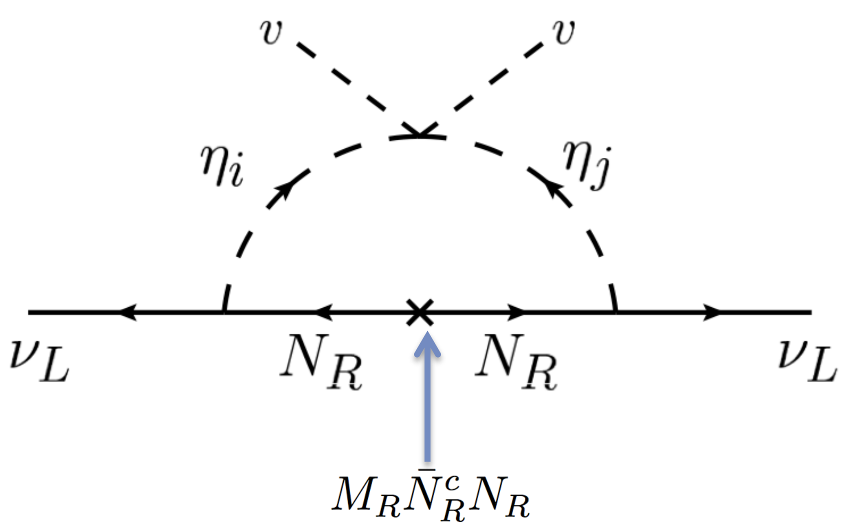
Boson	Φ	η
$(SU(2)_L, U(1)_Y)$	$(\mathbf{2}, 1/2)$	$(\mathbf{2}, 1/2)$
Z_2	$+$	$-$



Three requirements

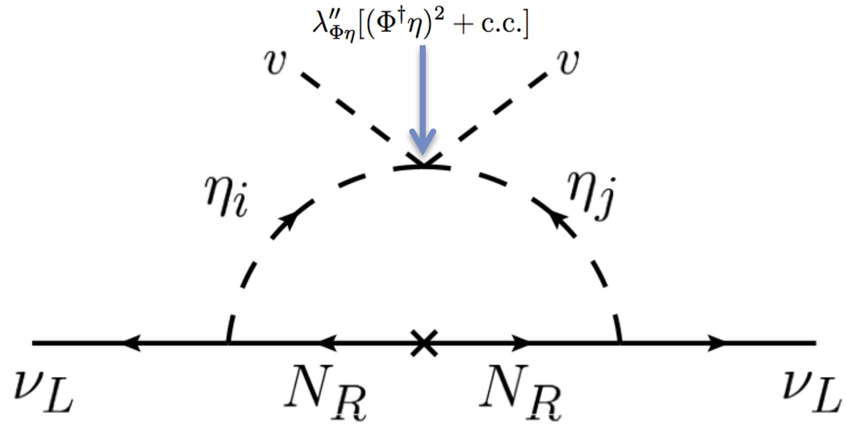
- Majorana mass
- $\lambda''_{\Phi\eta}$ is non-zero
- η is inert doublet

Majorana mass



If Majorana mass is forbidden,
this diagram can't draw

$\lambda''_{\Phi\eta}$ is non-zero

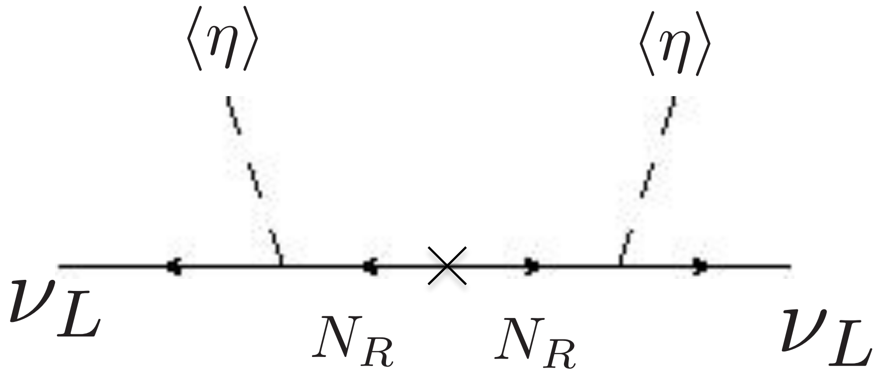


$$(\mathcal{M}_\nu)_{ab} = \frac{(y_\eta)_{ak}(y_\eta)_{bk}M_k}{(4\pi)^2}\left[\frac{m_R^2}{m_R^2 - M_k^2}\ln\frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2}\ln\frac{m_I^2}{M_k^2}\right]$$

$$m_R^2 - m_I^2 = 2\lambda''_{\Phi\eta}v^2$$

If $\lambda''_{\Phi\eta}$ is zero, neutrino masses are zero

η is inert doublet



If η has non-zero VEV, neutrino mass is generated by tree level

Three requirements

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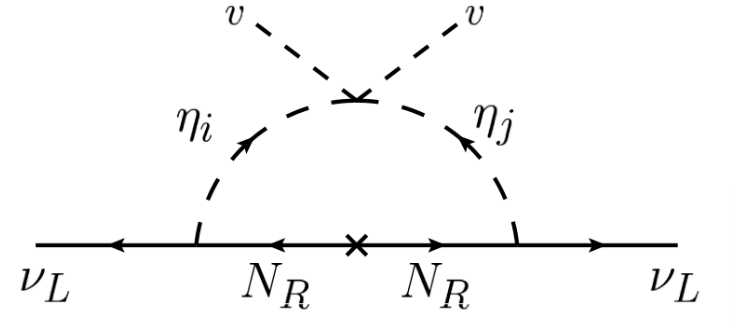
- η is inert doublet



Naturalness problem and Classically conformal symmetry

Naturalness problem

Hierarchy problem

$(126\text{GeV})^2$ ↓ m_0^2

$(2 \times 10^{18} GeV)^2$ ↓ Λ^2

$$m^2(\mu) = m_0^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left(m_Z^2 + 2m_W^2 + m^2 - 4m_t^2\right) + \mathcal{O}\left(\ln\frac{\Lambda}{\mu}\right)$$

We need about 30 digits fine-tuning

Classically conformal theory

Classically conformal theory
with no intermediate scale
can be an alternative solution
to the naturalness problem

Three requirements

- No mass term
- No intermediate scale
- CW type EWSB

Quadratic divergence

W.A. Bardeen, FERMILAB-CONF-95-391-T

Hierarchy problem

$$m^2(\mu) = m_0^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left(m_Z^2 + 2m_W^2 + m^2 - 4m_t^2\right) + \mathcal{O}\left(\ln\frac{\Lambda}{\mu}\right)$$

subtracted at UV scale
Once subtracted, no longer appears

Classically conformal invariance

Natural boundary condition is no mass terms at Planck scale

$$m^2(\Lambda) = m_0^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left(m_Z^2 + 2m_W^2 + m^2 - 4m_t^2\right) = 0$$

No intermediate scale

$$\delta m^2 \sim \lambda_{mix} m_{new}^2$$

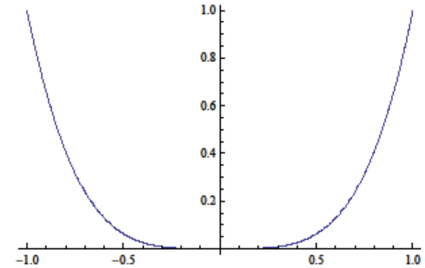
If large new scale exists,

Higgs mass has large correction.

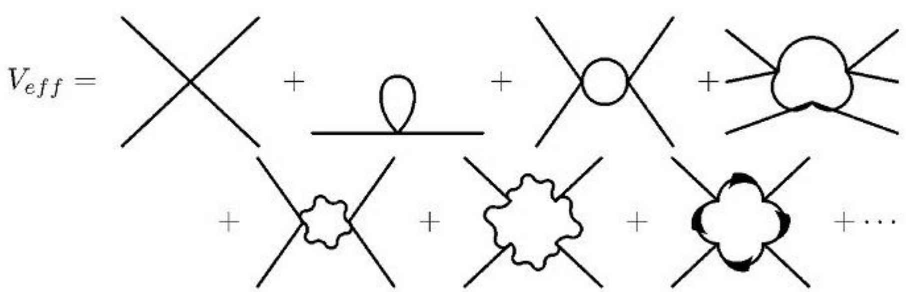
Coleman-Weinberg mechanism

Classically conformal potential

$$V(H) = \frac{\lambda}{4}\left(H^{\dagger}H\right)^{2} =$$



1-loop effective potential generates nonzero VEV



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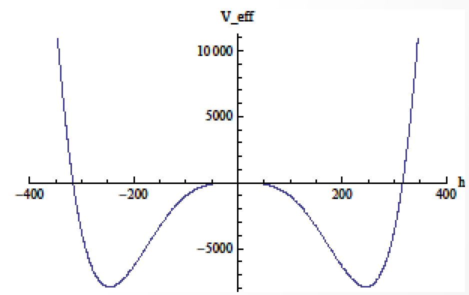

Coleman-Weinberg mechanism
(radiative symmetry breaking)

Three requirements

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Classically conformal B-L extended Ma model

Classically conformal Ma model

Ma model

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conflict

Classically conformal model

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- No intermediate scale
- CW type EWSB

Classically conformal B-L extended Ma model

Majorana mass term is forbidden by Classically conformal invariance

⬇ B-L gauged extension

Majorana mass is generated by B-L symmetry breaking

$$\frac{1}{2} y_N \varphi \bar{N}_R^c N_R \rightarrow M_R \bar{N}_R^c N_R$$

The classically conformal B-L extended Ma model

Gauge symmetry

$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)$

Fermion	L_L	e_R	N_R
$(SU(2)_L, U(1)_Y)$	$(\mathbf{2}, -1/2)$	$(\mathbf{1}, -1)$	$(\mathbf{1}, 0)$
$U(1)_{B-L}$	-1	-1	-1
Z_2	$+$	$+$	$-$

Boson	Φ	η	φ
$(SU(2)_L, U(1)_Y)$	$(\mathbf{2}, 1/2)$	$(\mathbf{2}, 1/2)$	$(\mathbf{1}, 0)$
$U(1)_{B-L}$	0	0	2
Z_2	$+$	$-$	$+$

DM candidate

Inert doublet

Lagrangian

Yukawa interaction

$$-\mathcal{L}_Y = (y_\ell)_a \bar{L}_{La} \Phi e_{Ra} + (y_\eta)_a \bar{L}_{La} \eta^* N_{Ra} + \frac{1}{2} y_N \varphi \bar{N}_R^c N_R + \text{h.c.}$$

Potential

$$\begin{aligned}\mathcal{V} = & \lambda_\Phi |\Phi|^4 + \lambda_\eta |\eta|^4 + \lambda_\varphi |\varphi|^4 + \lambda_{\Phi\eta} |\Phi|^2 |\eta|^2 + \lambda'_{\Phi\eta} |\Phi^\dagger \eta|^2 + \lambda''_{\Phi\eta} [(\Phi^\dagger \eta)^2 + \text{c.c.}] \\ & + \lambda_{\Phi\varphi} |\Phi|^2 |\varphi|^2 + \lambda_{\eta\varphi} |\eta|^2 |\varphi|^2,\end{aligned}$$

Classically conformal Ma model

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assumption

Lagrangian

Yukawa interaction

$$-\mathcal{L}_Y = (y_\ell)_a \bar{L}_{La} \Phi e_{Ra} + (y_\eta)_a \bar{L}_{La} \eta^* N_{Ra} + \frac{1}{2} y_N \varphi \bar{N}_R^c N_R + \text{h.c.}$$

Potential

non-zero

$$\mathcal{V} = \lambda_\Phi |\Phi|^4 + \lambda_\eta |\eta|^4 + \lambda_\varphi |\varphi|^4 + \lambda_{\Phi\eta} |\Phi|^2 |\eta|^2 + \lambda'_{\Phi\eta} |\Phi^\dagger \eta|^2 + \lambda''_{\Phi\eta} [(\Phi^\dagger \eta)^2 + \text{c.c.}]$$
$$+ \lambda_{\Phi\varphi} |\Phi|^2 |\varphi|^2 + \lambda_{\eta\varphi} |\eta|^2 |\varphi|^2,$$

We assume these couplings are zero at Planck scale for simplicity

Classically conformal Ma model

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CW mechanism

Effective potential

$$V_{eff}(\phi) = \frac{\lambda_\Phi(\phi)}{4}\phi^4$$

① $$\frac{dV_{eff}(\phi = M)}{d\phi} = 0$$ Minimum condition

② $$\frac{d^2V_{eff}(\phi = M)}{d\phi^2} > 0$$ Stability condition

CW mechanism

Effective potential

$$V_{eff}(\phi) = \frac{\lambda_\Phi(\phi)}{4}\phi^4$$

① $$\frac{dV_{eff}}{d\phi} = \frac{M^4}{4}\left(\frac{d\lambda_\Phi}{dt} + 4\lambda_\Phi\right) = 0$$

$$\Rightarrow \lambda_\Phi = -\frac{1}{4}\frac{d\lambda_\Phi}{dt} \sim -\frac{1}{64\pi^2}\left(96g'^4 - Y_N^4\right)$$

λ_Φ is generated by loop effect

② $$\frac{d^2V_{eff}}{d\phi^2} = -4\lambda_\Phi M^2 > 0 \quad \Rightarrow \quad \lambda_\Phi < 0$$

λ_Φ is negative

Electroweak symmetry breaking

If B-L symmetry is broken, the SM Higgs doublet mass is generated through the mixing term in the scalar potential.

$$V \sim \lambda_\Phi \Phi^4 + \lambda_{\Phi\varphi} \Phi^2 \varphi^2$$

⬇ Φ has VEV.

$$V \sim \lambda_\Phi \Phi^4 + \boxed{\lambda_{\Phi\varphi} \Phi^2 v_{B-L}^2}$$

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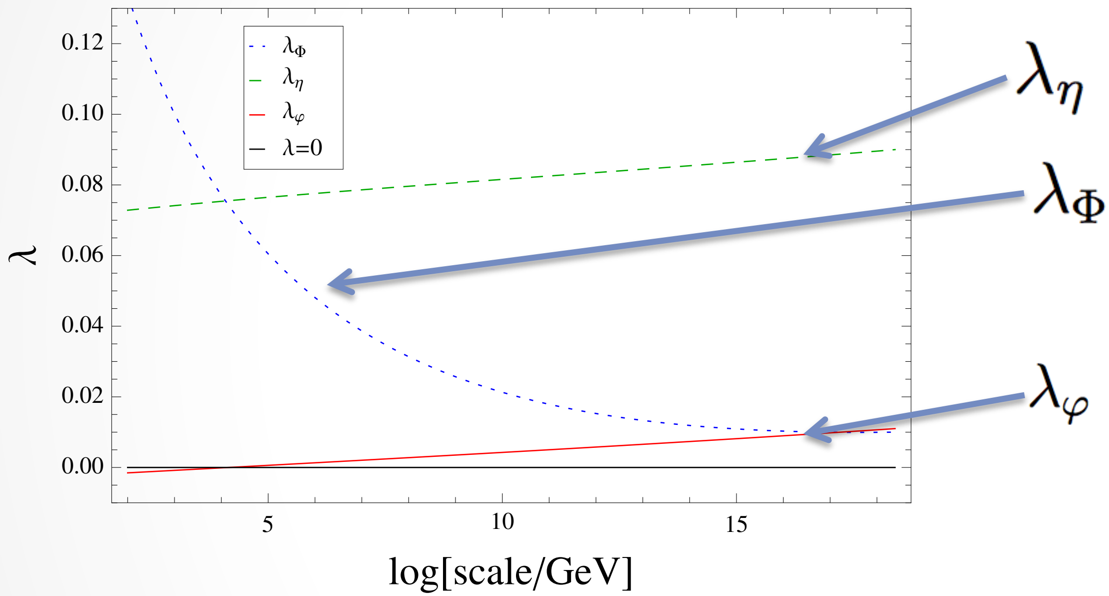
numerical analysis

Numerical result



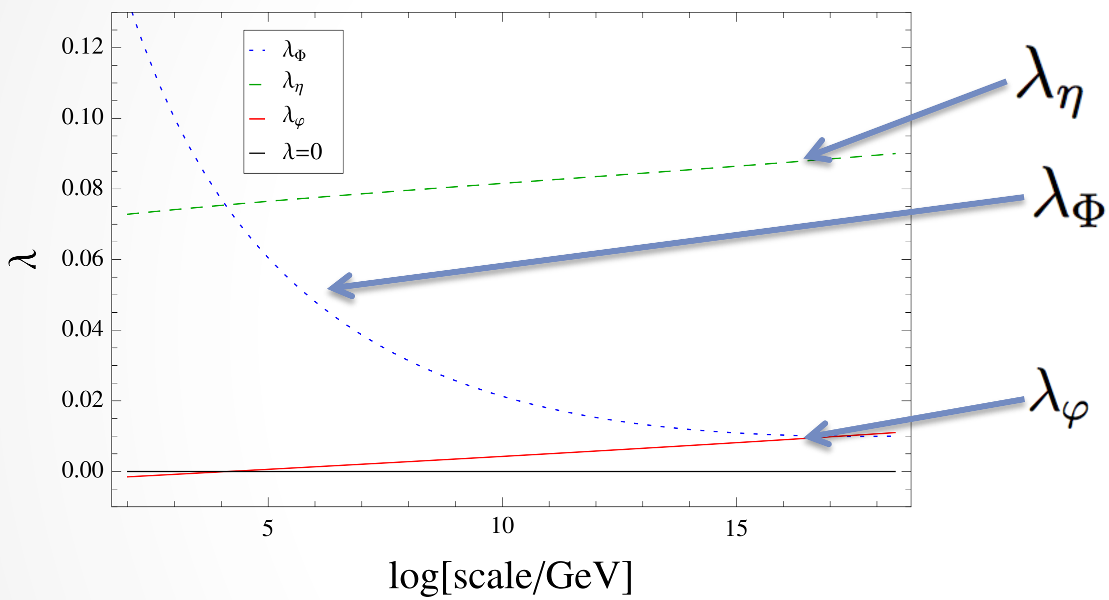
g_{B-L}=0.17

Y_N=0.2

$\lambda''_{\Phi\eta}$ =10^{-9}

$m_{z'}$=3.7 TeV

Numerical result



should be positive for stability

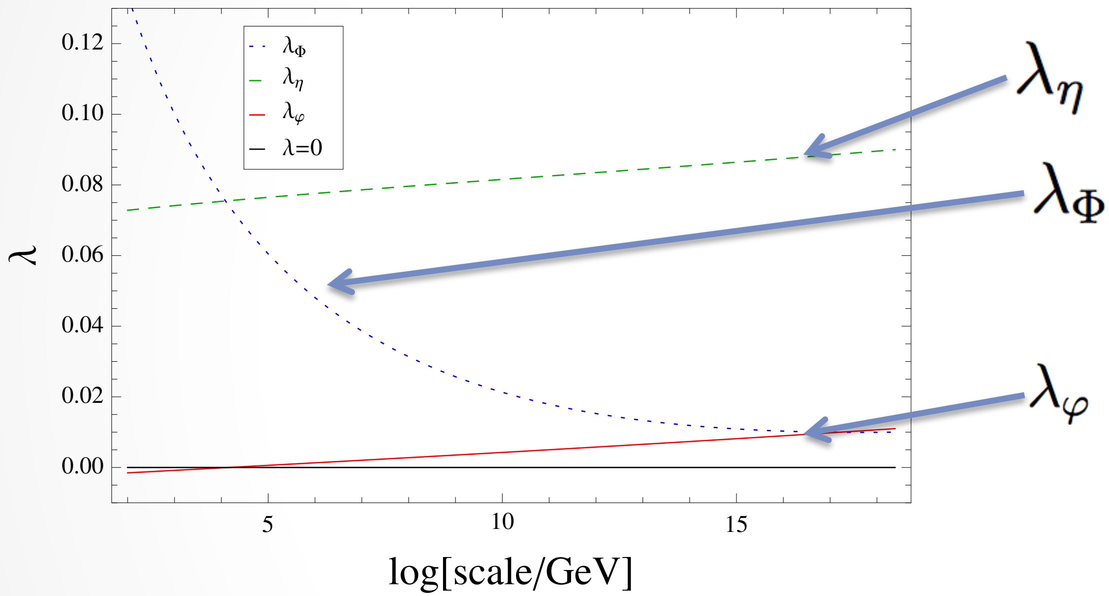
$g_{B\text{-}L}$=0.17

Y_N=0.2

$\lambda''_{\Phi\eta}=10^{-9}$

$m_{z'}$=3.7 TeV

Numerical result



should be positive for stability

$$\lambda_\Phi = -\frac{1}{4}\frac{d\lambda_\Phi}{dt} \sim -\frac{1}{64\pi^2}\left(96g'^4 - Y_N^4\right)$$

➡ $\lambda_\Phi < 0$

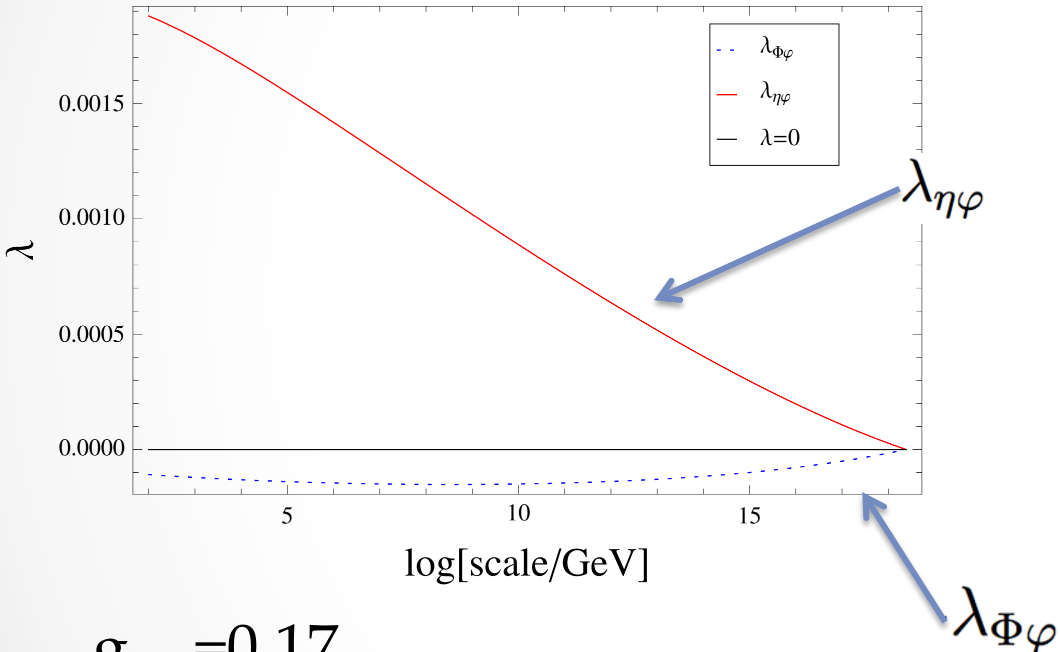
$g_{B\text{-}L}$=0.17

Y_N=0.2

$\lambda''_{\Phi\eta}$ =10^{-9}

$m_{z'}$=3.7 TeV

Numerical result



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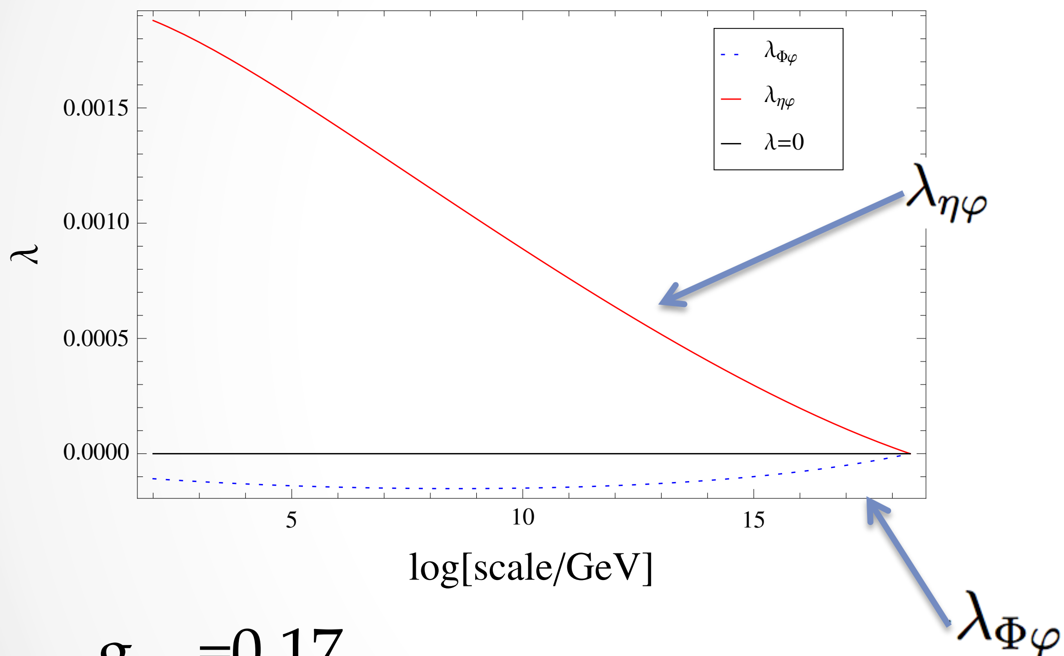
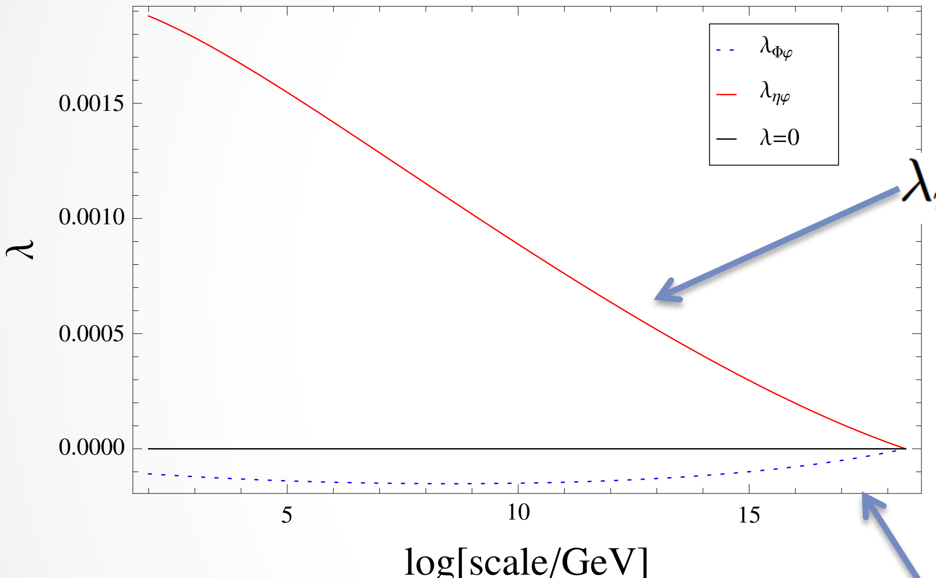


g_{B-L}=0.17
Y_N=0.2
$\lambda''_{\Phi\eta}=10^{-9}$
$m_{z'}$=3.7 TeV

Higgs mass term $\lambda_{\Phi\varphi} v_{B-L}^2 \Phi^2$

$\lambda_{\Phi\varphi} < 0$

Numerical result



$\lambda_{\eta\varphi}$

η mass term $\lambda_{\eta\varphi} v_{B-L}^2 \eta^2$

η is inert doublet

$\lambda_{\eta\varphi} > 0$

$\lambda_{\Phi\varphi}$

Higgs mass term $\lambda_{\Phi\varphi} v_{B-L}^2 \Phi^2$

$\lambda_{\Phi\varphi} < 0$

g_{B-L}=0.17
Y_N=0.2
$\lambda''_{\Phi\eta}=10^{-9}$
$m_{Z'}$=3.7 TeV

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② EW(TeV) scale and Planck scale
⇒Classically conformal symmetry

Back up

Yukawa coupling

$$(\mathcal{M}_\nu)_{ab} = \frac{(y_\eta)_{ak}(y_\eta)_{bk}M_k}{(4\pi)^2}\left[\frac{m_R^2}{m_R^2-M_k^2}\ln\frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2-M_k^2}\ln\frac{m_I^2}{M_k^2}\right]$$

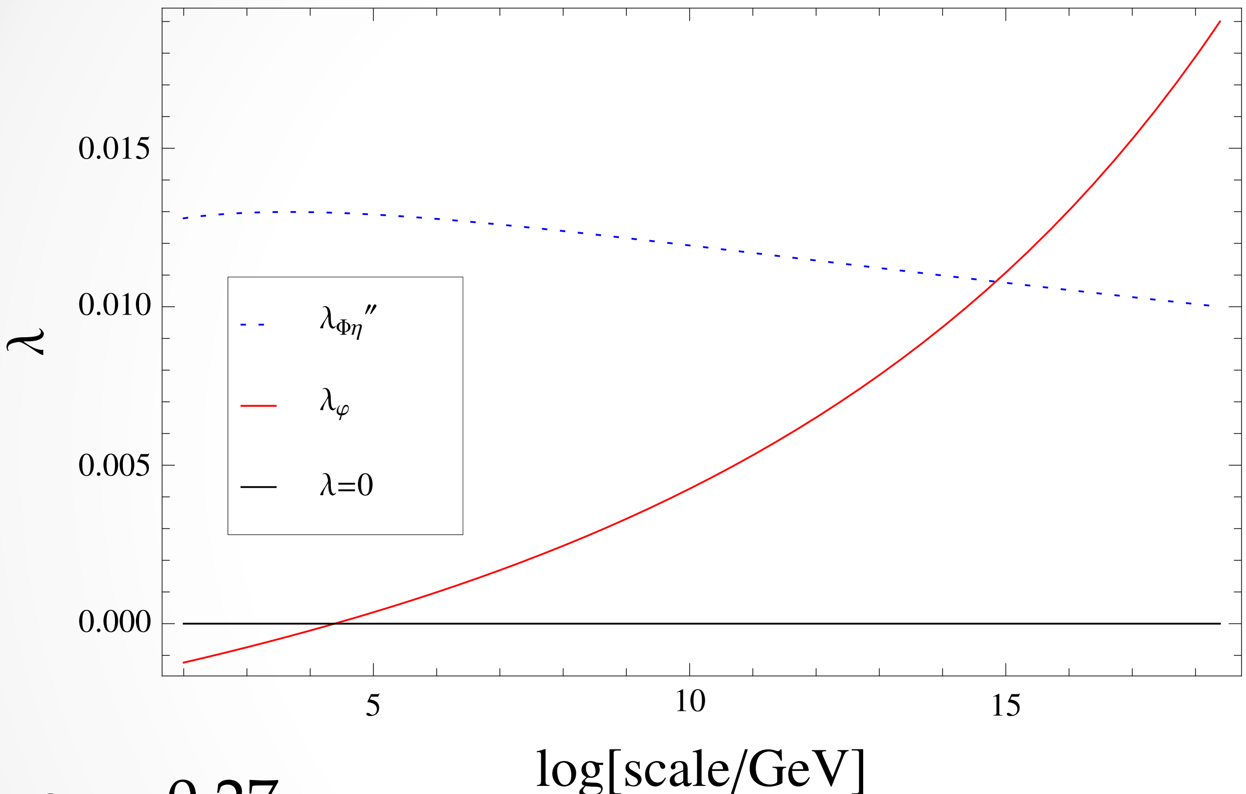
$$Y_\eta = U^*_{MNS}\begin{pmatrix} m_1^{1/2} & 0 & 0 \\ 0 & m_2^{1/2} & 0 \\ 0 & 0 & m_3^{1/2} \end{pmatrix} O R^{-1/2}$$

$$O = \begin{pmatrix} 0 & 0 & 1 \\ \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \end{pmatrix} \qquad R_i = M_i\left[\frac{m_R^2}{m_R^2-M_i^2}\ln\frac{m_R^2}{M_i^2} - \frac{m_I^2}{m_I^2-M_i^2}\ln\frac{m_I^2}{M_i^2}\right]$$

Normal hierarchy

$m_1=0$

$\alpha \neq 0$ case



$g_{B-L}=0.27$

$Y_N=0.5$

$\lambda''_{\Phi\eta}=10^{-2}$

$\alpha=9i$

$$(4\pi)^2\frac{dg'}{dt} = 7g'^3 \qquad (4\pi)^2\frac{dg}{dt} = -3g^3 \qquad (4\pi)^2\frac{dg_3}{dt} = -7g_3^3$$

$$(4\pi)^2\frac{dg_{B-L}}{dt} = g_{B-L}\left(12g_{B-L}^2 + \frac{32}{3}g_{B-L}g_{mix} + 7g_{mix}^2\right)$$

$$(4\pi)^2\frac{dg_{mix}}{dt} = 12g_{B-L}^2 g_{mix} + \frac{32}{3}g_{B-L}\left(g_{mix}^2 + g'^2\right) + 7g_{mix}\left(g_{mix}^2 + 2g'^2\right)$$

$$(4\pi)^2\frac{\lambda_\Phi}{dt} = 24\lambda_\Phi^2 + 2\lambda_{\Phi\eta}^2 + \lambda_{\Phi\eta}'^2 + 4\lambda_{\Phi\eta}''^2 + 2\lambda_{\Phi\eta}\lambda_{\Phi\eta}' + \lambda_{\Phi\varphi}^2$$
$$+ \frac{3}{8}\left[2g^4 + \left(g^2 + g'^2 + g_{mix}^2\right)^2\right] - 3\lambda_\Phi\left[3g^2 + g'^2 + g_{mix}^2\right] - 6y_t^4 + 12\lambda_\Phi y_t^2$$

$$(4\pi)^2\frac{\lambda_\eta}{dt} = 24\lambda_\eta^2 + 2\lambda_{\Phi\eta}^2 + \lambda_{\Phi\eta}'^2 + 4\lambda_{\Phi\eta}''^2 + 2\lambda_{\Phi\eta}\lambda_{\Phi\eta}' + \lambda_{\eta\varphi}^2 + \frac{3}{8}\left[2g^4 + \left(g^2 + g'^2 + g_{mix}^2\right)^2\right]$$

$$- 3\lambda_\eta\left[3g^2 + g'^2 + g_{mix}^2\right] - 2Tr\left[y_\eta^\dagger y_\eta y_\eta^\dagger y_\eta\right] + 4\lambda_\eta Tr\left[y_\eta^\dagger y_\eta\right]$$

$$(4\pi)^2\frac{\lambda_\varphi}{dt} = 20\lambda_\varphi^2 + 2(\lambda_{\Phi\varphi}^2 + \lambda_{\eta\varphi}^2) + 96g_{B-L}^4 - 48\lambda_\varphi g_{B-L}^2 - Tr\left[y_N^\dagger y_N y_N^\dagger y_N\right] + 2\lambda_\varphi Tr\left[y_N^\dagger y_N\right]$$

$$(4\pi)^2\frac{\lambda_{\Phi\eta}}{dt} = \lambda_{\Phi\eta}\left[4\lambda_{\Phi\eta} + 12\lambda_\Phi + 12\lambda_\eta + 2Tr\left[y_\eta^\dagger y_\eta + y_\ell^\dagger y_\ell\right] - 3\left(3g^2 + g'^2 + g_{mix}^2\right) + 6y_t^2\right]$$
$$+ 2\lambda_{\Phi\varphi}\lambda_{\eta\varphi} + 4\lambda_\eta\lambda_{\Phi\eta}' + 4\lambda_\Phi\lambda_{\Phi\eta}' + 2\lambda_{\Phi\eta}'^2 + 8\lambda_{\Phi\eta}''^2 + \frac{3}{4}\left(2g^4 + \left(g^2 - g'^2 - g_{mix}^2\right)^2\right) - 4Tr\left[y_\eta^\dagger y_\eta y_\ell^\dagger y_\ell\right]$$

$$(4\pi)^2\frac{\lambda_{\Phi\eta}'}{dt} = \lambda_{\Phi\eta}'\left[4\lambda_\Phi + 4\lambda_\eta + 8\lambda_{\Phi\eta} + 4\lambda_{\Phi\eta}' + 2Tr\left[y_\eta^\dagger y_\eta + y_\ell^\dagger y_\ell\right] + 6y_t^2 - 3\left(3g^2 + g'^2 + g_{mix}^2\right)\right]$$
$$+ 16\lambda_{\Phi\eta}''^2 + 3g^2\left(g'^2 + g_{mix}^2\right) + 4Tr\left[y_\eta^\dagger y_\eta y_\ell^\dagger y_\ell\right]$$

$$(4\pi)^2\frac{\lambda_{\Phi\eta}''}{dt} = 4\lambda_{\Phi\eta}''\left[\lambda_\Phi + \lambda_\eta + 2\lambda_{\Phi\eta} + 3\lambda_{\Phi\eta}' + \frac{1}{2}Tr\left[y_\eta^\dagger y_\eta + y_\ell^\dagger y_\ell\right] + \frac{3}{2}y_t^2 - \frac{3}{4}\left(3g^2 + g'^2 + g_{mix}^2\right)\right]$$

mixing

$$(4\pi)^2 \frac{\lambda_{\Phi\varphi}}{dt} = 4\lambda_{\Phi\varphi}^2 + 12\lambda_{\Phi\varphi}\lambda_{\Phi} + (4\lambda_{\Phi\eta} + 2\lambda'_{\Phi\eta})\lambda_{\eta\varphi} + 8\lambda_{\Phi\varphi}\lambda_{\varphi} + 12g_{mix}^2 g_{B-L}^2$$
$$+ \lambda_{\Phi\varphi}\left[6y_t^2 + Tr\left[y_N^\dagger y_N\right] - \frac{3}{2}\left(3g^2 + g'^2 + g_{mix}^2\right) - 24g_{B-L}^2\right],$$

$$(4\pi)^2 \frac{\lambda_{\eta\varphi}}{dt} = 4\lambda_{\eta\varphi}^2 + 12\lambda_{\eta\varphi}\lambda_{\eta} + (4\lambda_{\Phi\eta} + 2\lambda'_{\Phi\eta})\lambda_{\Phi\varphi} + 8\lambda_{\eta\varphi}\lambda_{\varphi} + 12g_{mix}^2 g_{B-L}^2 - 4Tr\left[y_\eta^\dagger y_\eta y_N^\dagger y_N\right]$$
$$+ \lambda_{\eta\varphi}\left[6y_t^2 + Tr\left[y_N^\dagger y_N\right] - \frac{3}{2}\left(3g^2 + g'^2 + g_{mix}^2\right) - 24g_{B-L}^2\right].$$

Yukawa

$$(4\pi)^2\frac{dy_\eta}{dt} = y_\eta\left[\frac{3}{2}y_\eta^\dagger y_\eta + \frac{1}{2}y_\ell^\dagger y_\ell + Tr\left[y_\eta^\dagger y_\eta\right] - \frac{3}{4}\left(g'^2 + g_{mix}^2\right) - \frac{9}{4}g^2 - 6g_{B-L}^2 - 3g_{B-L}g_{mix}\right]$$

$$(4\pi)^2\frac{dy_\ell}{dt} = y_\ell\left[\frac{3}{2}y_\ell^\dagger y_\ell + \frac{1}{2}y_\eta^\dagger y_\eta + Tr\left[y_\ell^\dagger y_\ell\right] - \frac{15}{4}\left(g'^2 + g_{mix}^2\right) - \frac{9}{4}g^2 - 6g_{B-L}^2 - 9g_{B-L}g_{mix}\right]$$

$$(4\pi)^2\frac{dy_t}{dt} = y_t\left[\frac{9}{2}y_t^2 - 8g_3^2 - \frac{9}{4}g^2 - \frac{17}{12}\left(g'^2 + g_{mix}^2\right) - \frac{2}{3}g_{B-L}^2 - \frac{5}{3}g_{mix}g_{B-L}\right]$$

$$(4\pi)^2\frac{dy_N}{dt} = y_N\left[y_N^\dagger y_N + \frac{1}{2}Tr\left[y_N^\dagger y_N\right] - 6g_{B-L}^2\right]$$